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ISTITUTO IDROGRAFICO DELLA MARINA

NONLINEAR OSCILLATIONS OF MEAN ANNUAL SEA LEVEL DATA AT GENOA

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Preface

The study of tides is crucial for the Istituto Idrografico della Marina, the Italian official charting agency since 1872, as well as a leading scientific research and training centre. In addition to nautical documents, the IIM issues a number of scientific publications, actively contributing to marine science.
The tide gauge of Genoa is the starting point of the Italian levelling network and boasts one of the oldest uninterrupted monitoring in the whole Mediterranean Sea. Consequently, data collected in Genoa by the IIM allow scientists to study mean sea level oscillations in time, as well as in space comparing data from different tide gauges throughout Italy.
Understanding mean sea level variations is important to assess the extent of climate change and global warming.
"Nonlinear oscillations of mean annual sea level data in Genoa" is the outcome of one of these studies.

Istituto Idrografico della Marina
Captain Luigi SINAPI
Director

Introduction

In the present paper we examined the tide gauge records of Genoa (44°25’ N, 08°54’ E) from 1928 to 2016 to investigate long period oscillations of mean annual sea level data. We found a significant oscillation with a period of 18.6 years associated with the period of the retrograde orbital motion of the Moon’s nodes through a complete cycle: the Moon’s nodes are the points where the Moon’s orbit intercepts the Earth celestial ecliptic (Schureman,1941). Although the tide gauge data of Genoa are limited, the mean annual sea level rise can hypothetically be described in terms of harmonic and subharmonic oscillations of the 18.6 year tidal wave.

Harmonic analysis

In signal processing the Finite Impulse Response (FIR) filter is a filter whose impulse response is of finite duration. The Matlab FIR1 filter has been used with a Hanning window centred at angular velocity: 0.3376 rad/yr (period=18.61 years) and a bandwidth of 0.104 rad/yr. We show in fig.1 the mean annual sea level data in Genoa and the output of the FiR filter corresponding to the first harmonic with a period of 18.61 years.

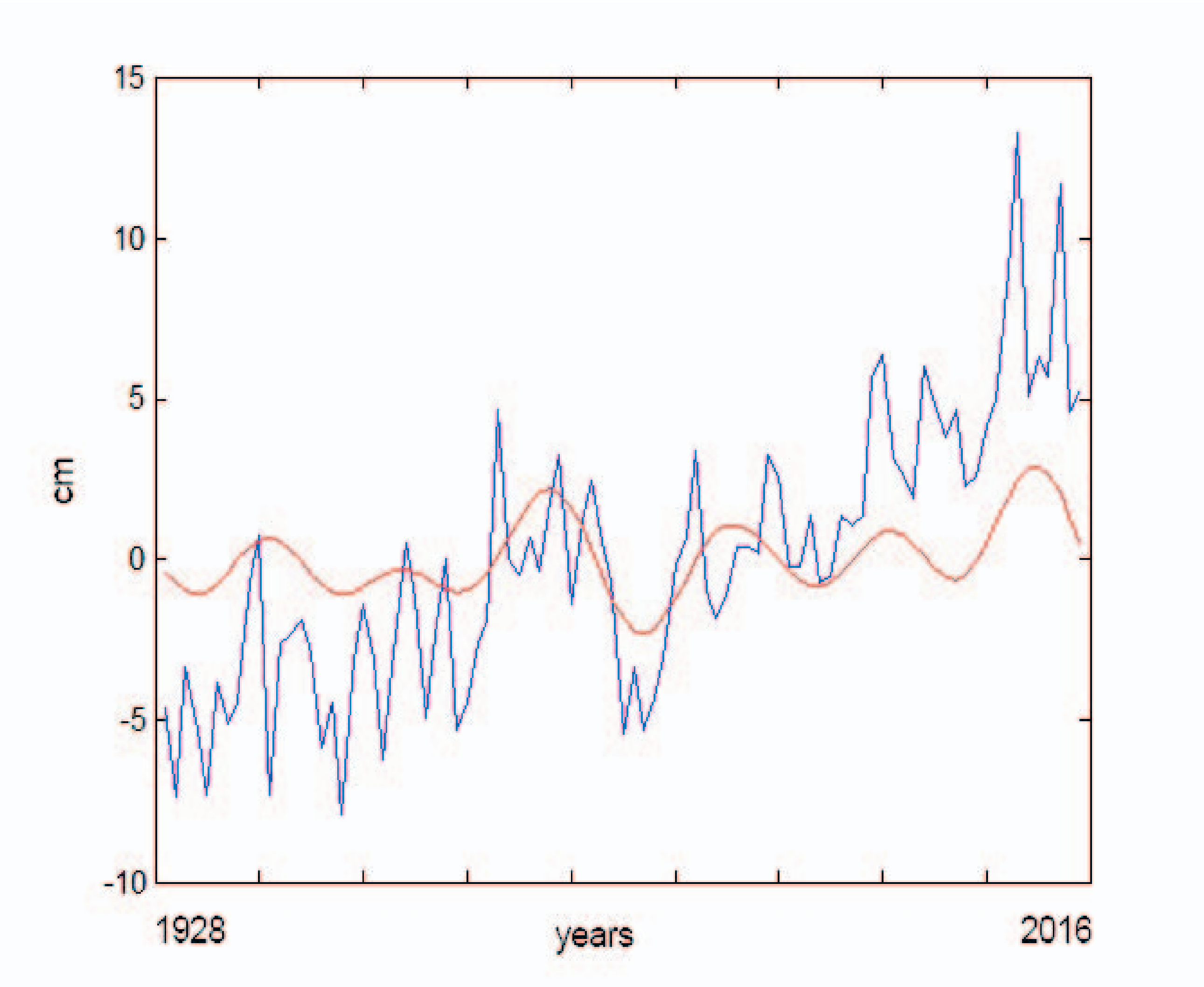

Fig.1 Mean annual sea level data in Genoa (blue) and long period oscillation of 18.6 years (red)

Having established four components in the harmonic analysis of the mean sea level data, we computed three FIR filters centred at the angular velocities: 0.6756 rad/yr (period=9.3 years), 1.0134 rad/yr (period=6.2 years), 1.3659 rad/yr (period=4.6 years).
The superposition of the four filter outputs corresponding to the first four harmonics of the Matlab computation is plotted in fig.2.

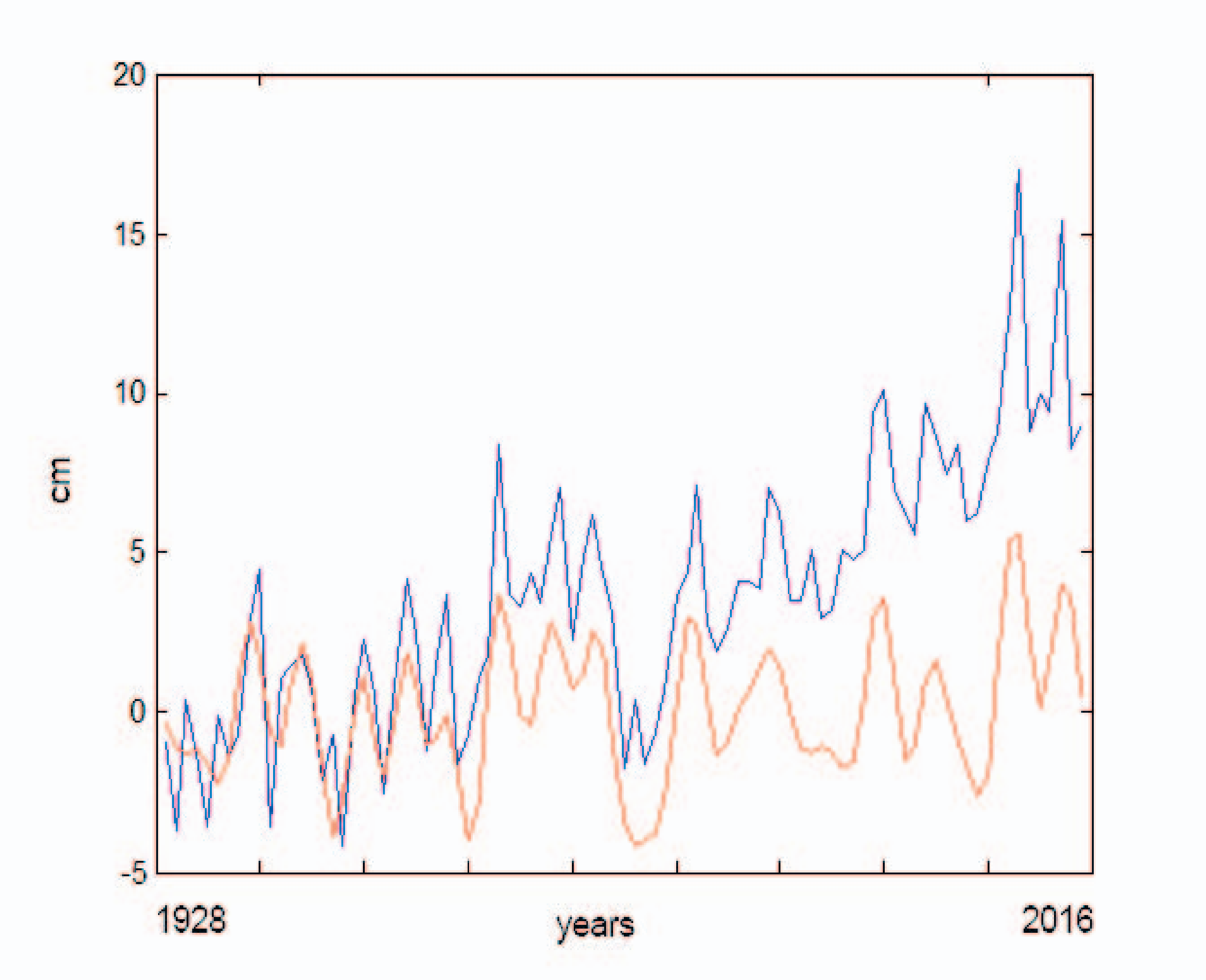

Fig.2 Mean annual sea level data in Genoa (blue) and the superposition of four harmonics (red)

Subharmonic analysis

It is well known that when a periodic force is applied to a linear system the resulting motion is obtained by a superposition of the transient and the steady state solution. Thus, as far as linear systems are concerned, the forced oscillation is determined once the system and the external forces are given, and is by no means affected by the initial condition with which the oscillation was started. The nonlinear systems, however, can possess a wide variety of periodic oscillations in addition to those which have the same period as the external force. It has been pointed out by Trefftz (1926) that if the solution of a differential equation:

$$d^2v / dt^2 = F(v, dv/dt, t) \quad \text{with} \quad F(v, dv/dt, t+T) = F(v, dv/dt, t) \qquad (1)$$

is stable, it must be a periodic solution in which the least period is equal to the period T of the external force, or to an integral multiple of T.

Corresponding to these two cases the terms “harmonic” and “subharmonic” oscillations are respectively applied. Contrary to many cases of linear differential equations, it is hardly possible to find the general solution of equation (1) for the given initial conditions. A conventional method of solution is to assume for v(t) a Fourier series development with undetermined coefficients, and then to fix them by nonlinear relations obtained by substituting the series into the equation (1). It should, however, be noticed that this method of solution is merely to find out the periodic states of equilibrium, which are not always sustained, but last so long as they are stable. The circumstances under which this condition occurs are determined by the stability of the periodic states of equilibrium. The study of the periodic solutions is closely related to the investigation of the stability of the periodic states of equilibrium correlated with singular points. We now deal with the subharmonic oscillations whose frequencies are a fraction 1/2, 1/3, ..., of the frequency of the external force.

As a typical example of analysis of subharmonic oscillations by means of integral curves, we treat the subharmonic oscillation of order 1/3. Consider the following equation:

$$d^2v/dt^2 + k\,dv/dt + v^3 = B\cos(3t) \qquad (2)$$

where the restoring force is expressed by a cubic nonlinear function of v and the argument of the external force is 3t. The solution of the equation (2) is assumed of the form:

$$v(t) = x(t)\sin(t) + y(t)\cos(t) + W\cos(3t) \qquad (3)$$

where the amplitudes x(t) and y(t) are both functions of t in the transient state, but are reduced to constants in the steady state. Following Mandelstam and Papalexi (1932), the amplitude W may be assumed to be:

$$W = 1/(1-3^2)\,B = -0.125\,B \qquad (4)$$

From the equation (2), and taking into account (3) and (4), we get:

$$dx/dt = 1/2\,[-k\,x + A\,y + 3/4\,W\,(x^2 - y^2)] = X(x,y)$$

$$dy/dt = -1/2\,[A\,x + k\,y + 3/4\,W\,2\,x\,y] = Y(x,y) \qquad (5)$$

where: $A = 1 - 3/4\,(x^2 + y^2) - 3/2\,W^2$. Equations (5) play a significant role in the study of the transient state and the steady state. The oscillations in the steady state are correlated with the singular points determined by:

$$X(x, y) = 0, \quad Y(x, y) = 0 \qquad (6)$$

and the transient solution can be obtained by the integral curves of:

$$dy/dx = Y(x, y) / X(x, y) \qquad (7)$$

where X(x, y) and Y(x, y) are given by equations (5). Since time t does not occur explicitly in equation (7) we can draw integral curves in the x–y plane with the aid of the isocline method. Periodic solutions are related with x(t)=constant, y(t)=constant of equations (5) and therefore with the singular points of equation (7), i.e. the points at which X(x, y) and Y(x, y) both vanish. According to the initial conditions an oscillation can show a subharmonic character or on the contrary no subharmonic oscillation takes place if the final singular point is a stable spiral point.

At present our understanding of the mechanism responsible for the development of subharmonic waves in the mean annual sea level values in Genoa is quite limited; nevertheless we make the hypothesis that nonlinear effects on forcing sea level variations can produce oscillations whose frequencies are a fraction 1/3,1/5 of the frequency of the external force of the retrograde orbital motion of the Moon's nodes (T=18.61 years). The superposition of four harmonics and two subharmonic waves of order 1/3, 1/5 is plotted in fig. 3.

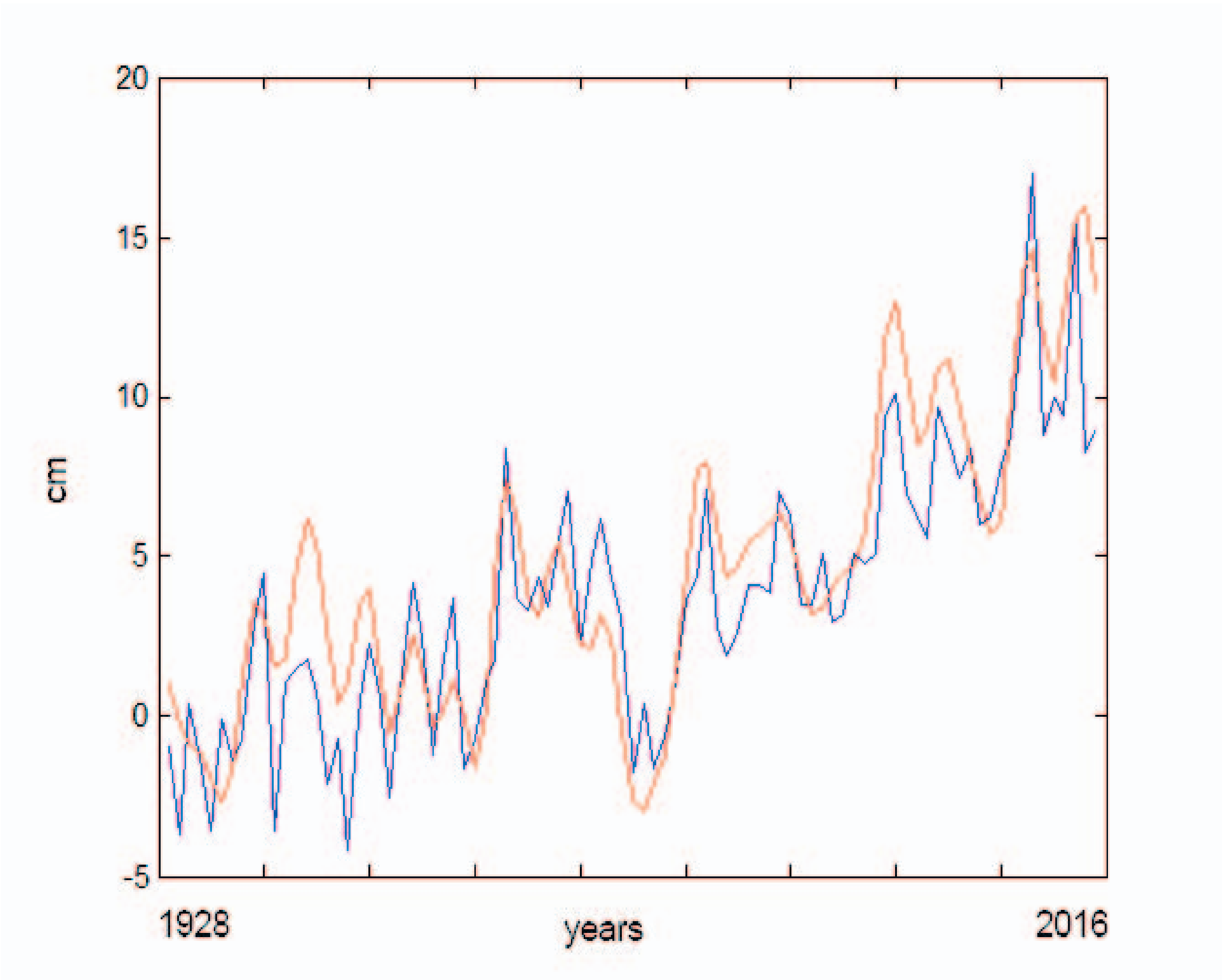

Fig.3 Mean annual sea level data in Genoa (blue)
and the superposition of four harmonics and two subharmonic waves (red)

Conclusions

At the Thomson gauge of Genoa a permanent sea land reference mark is used to measure sea level changes (Lusetti, 1977). A statistical investigation on the mean annual data from 1928 to 2006 has produced a linear positive trend of about 1.1 mm per year (Demarte et at., 2007). In the present paper the mean sea level from 2007 to 2016 has shown a positive rate of 10.7 cm compared to the 1937 to 1946 average value which is the standard reference mark for the Italian terrestrial topography. Over the last decade, there has been a significant progress in understanding future sea level values as a result of satellite and in "situ" observations and improved models (Chambers et al., 2012). As a result, there is a huge demand for projections of sea level scenarios, particularly at local and regional level. The investigation of the harmonic and the subharmonic oscillations of the 18.6 year tidal wave may play an important role to predict sea level changes in Genoa over coming decades.

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